# THE DYNAMIC PLASTIC BEHAVIOR OF FULLY CLAMPED **RECTANGULAR PLATES**

### NORMAN JONES, T. O. URAN and S. A. TEKIN

Massachusetts Institute of Technology, Department of Naval Architecture and Marine Engineering, Cambridge, Mass. 02139

Abstract—An experimental investigation was undertaken in order to study the behavior of fully-clamped rectangular plates when subjected to uniformly distributed impulsive velocities. The total energy of the dynamic loads was sufficiently large to cause plastic flow of the plate material and maximum permanent deflections from 0.2 to nearly seven times the corresponding plate thicknesses. All the rectangular plates had the same aspect ratio (B/L = 0.593) but various thicknesses (H), and were made from either hot-rolled mild steel or aluminum 6061-T6. The permanent deformed profiles of the plates are similar to the shape of the velocity field used by Wood [11] for calculating the minimum upper bound to the collapse pressure of a fully-clamped rectangular plate loaded with a uniformly distributed time-independent pressure. It is observed that a modification of the bending only prediction of Martin [13] provides adequate engineering estimates of the maximum permanent deflections up to the order of one-half of the corresponding plate thickness. For larger deflections, it is necessary to include the influence of geometry changes; and in the case of mild steel, material strain-rate sensitivity as well.

### NOTATION

- В semi-width of plate
- thickness of plate Н
- semi-length of plate L
- $M_0 = \sigma_0 H^2/4$
- $\frac{V_0}{W}$ initial velocity of plate
- transverse deflection
- $W_0$ permanent transverse deflection at center of plate
- mass/unit surface area of plate т
- duration of response  $t_f$
- x, ycartesian coordinates defined in Figs. 3 and 5
- coordinate defined in Fig. 5 x
- β B/L
- $\delta_r$ permanent transverse deflection at center of plate

$$\lambda = \rho \frac{V_0^2 L^2}{M_0}$$

- density of specimens ρ
- yield stress in simple tension  $\sigma_0$
- ultimate tensile stress  $\sigma_{u}$

### **INTRODUCTION**

It is often necessary to estimate the maximum dynamic energy which a structure could absorb without failure, or to predict the damage which a structure would sustain if it were involved in a collision with another body or subjected to explosive loads. The numerical procedures which have been developed by several authors [1-3, etc.] can be used in order to predict the response of a variety of structures when subjected to dynamic loads. However, there is a paucity of experimental results with which these numerical estimates can be compared since most of the experimental programmes have been confined to the study of beams [4–8, etc.], axisymmetric plates [9, etc.], or single tests on various other structures rather than parametric studies.

It is well known that approximate methods or formulae are powerful tools in the initial design phase of most engineering projects. Such simplified procedures allow unsuitable designs to be eliminated quickly and give the approximate dimensions or behavior of the final design (or designs). These results may then be fed as input into a computer programme in order to predict the behavior more accurately and to examine the influence of various details which are disregarded in approximate methods. Theoretical investigations of the dynamic plastic response of the structures are simplified considerably when the influence of material elasticity is disregarded. Rigid–plastic analyses, as they are known, are believed to be valid provided the dynamic energy is considerably larger than the maximum energy which could be absorbed in a wholly elastic manner. Moreover, the duration of loading should be short compared with the fundamental period of vibration [4].

The authors present the results of over 40 tests on rectangular plates which were subjected to uniformly distributed impulsive velocities with an initial energy considerably larger than the maximum elastic strain energy capacity of the corresponding structure. The rectangular plates, which were fully clamped around the outer boundary, were made either from mild steel or aluminum 6061-T6. A comparison of these results permits an estimate of the influence of strain-rate sensitivity to be made, since mild steel is very sensitive to strain-rate while aluminum 6061-T6 is not. It is hoped that these results will aid in the further development of approximate rigid–plastic methods, particularly for the finite-deflections of non-axisymmetric structures, and enable the accuracy of the numerical procedures (or of the experiments) to be assessed.

## EXPERIMENTAL ARRANGEMENT

A ballistic pendulum was employed in order to measure the dynamic energy imparted to the rectangular plates by sheet explosive. The experimental arrangement was similar to that used for the experiments reported in Ref. [10] so an interested reader is referred there for details. In order to assess the windage and friction losses of the system, the ballistic pendulum was released from various initial amplitudes and allowed to swing freely over a number of cycles as indicated in Fig. 1. It is evident that the loss per quarter cycle during the initial few swings was less than 0.5 per cent of the corresponding amplitude. In view of the small value of this loss, no corrections were made to the results reported herein.

The device, which was attached to the pendulum head and used for supporting the specimens, is indicated in Fig. 2. The gripping surfaces were serrated and high tensile strength steel bolts were employed in an attempt to ensure that a fully clamped support condition with no axial movement would be achieved. A typical rectangular plate specimen is indicated in Fig. 3. The target areas of all specimens measured  $3 \text{ in.} \times 5\frac{1}{16}$  in. and the various plate thicknesses are indicated in Tables 1 and 2. The specimens were made from either hot-rolled mild steel or aluminum 6061-T6 as received from the supplier and had the average mechanical and chemical properties listed in Tables 3 and 4. Stress-strain curves of two typical tensile specimens which were cut from the same mild steel and aluminum 6061-T6 sheets as some of the plates are shown in Fig. 4. These tests were



FIG. 1. Reduction in maximum amplitude of the ballistic pendulum ( $\Delta$ ) over the first few free oscillations (N).



FIG. 2. Side elevation and plan view of the device used for clamping the rectangular plates to the ballistic pendulum.



FIG. 3. Rectangular plate specimens.

conducted on an Instron testing machine at average strain rates of approximately  $5 \times 10^{-4}$  in./in./sec.

In order to prevent spalling, a neoprene or foam attenuator was located between the sheet explosive and the surface of each specimen. It was observed by the authors of Ref. 10 and here that foam attenuators disintegrated completely, while neoprene attenuators generally remained fairly intact but were blown some distance away from the pendulum.

Specimen No.	Н (in.)	V <sub>0</sub> (ft/sec)	Attenuator foam (F) neoprene (N)	W <sub>o</sub> (in.)	$W_0/H$	Â
1	0-0643	134-08	F	0.2278	3.542	322
2	0.0644	152-60	F	0.2654	4.120	437
3	0.0638	165.06	F	0.2966	4.650	521
4	0.0638	180.06	F	0.3296	5.166	619
5	0.0647	233.00	F	0.4155	6.420	1010
6	0.0635	234.08	F	0.4270	6.730	1058
7	0.0998	80.73	F	0.1068	1.046	52.7
8	0.0985	118-95	F	0.1864	1.890	123-1
9	0.0984	124-20	F	0.1917	1.940	134-1
10	0.0983	161-70	F	0.2700	2.755	228.2
11	0.0982	177.90	N	0.3278	3.330	276.3
12	0.0982	202.60	F	0.3695	3.760	359
13	0.0983	216.70	F	0.4068	4.135	408
14	0.0984	231.13	F	0.4235	4.300	465
15	0.1728	69.69	F	0.0535	0.310	12.92
16	0.1728	88-85	F	0.0890	0.515	20.92
17	0.1725	130-90	F	0.1764	1.022	45-6
18	0.1729	153-36	F	0.2175	1.257	62.25
19	0.1728	166-26	F	0.2440	1.411	73.25
20	0.1729	165-70	Ν	0.2455	1.420	72.7
21	0-1731	171.10	F	0.2734	1.580	77.4
22	0.1727	178.02	F	0-2963	1.715	84

TABLE 1. DATA FOR HOT-ROLLED MILD STEEL SPECIMENS

Specimen No.	$\begin{array}{c} H & V_0 \\ \text{(in.)} & (\text{ft/sec}) \end{array}$		Attenuator foam (F) neoprene (N)	<i>W</i> <sub>0</sub> (in.)	$W_0/H$	ړ
1	0.244	190.44	F	0.109	0.446	13.88
$\tilde{2}$	0.244	148-92	F	0.047	0.192	8-48
3	0.244	346-88	F	0.325	1.330	45.84
4	0.244	297-13	F	0.225	0.920	33.85
5	0.244	275-29	F	0.214	0.870	29
6	0.244	331-59	F	0.297	1.211	42
7	0.188	259.71	Ν	0.227	1.205	44.25
8	0.188	226.56	F	0.192	1.025	33.72
9	0.188	183-01	Ν	0.158	0.843	22
10	0.188	395-32	F	0.376	2.000	102-4
11	0.188	417.87	F	0.454	2.410	114-64
12	0.188	176-96	F	0.151	0.805	20.56
13	0.189	201.57	F	0.158	0.837	26-6
14	0.188	412.40	F	0.418	2.220	111-5
15	0.122	267.95	N	0.289	2.362	110.8
16	0.122	251.86	F	0.271	2.205	97.9
17	0.123	330.42	N	0.350	2.842	167.8
18	0.123	230.99	F	0.222	1.820	82.25
19	0.123	389-20	F	0.429	3.480	233.4

TABLE 2. DATA FOR ALUMINUM 6061-T6 SPECIMENS

In view of the fact that Humphreys [6] used sponge rubber, and Florence and Firth [7] and the authors of Ref. [8] used neoprene, it was decided to assess the influence, if any, of different attenuator materials on the results recorded previously. It appears from the results, which are presented in Tables 1 and 2 and Fig. 6, that the type of attenuator did not influence the outcome of the test in any way except for the fact that a  $\frac{1}{8}$  in. thick layer of neoprene allowed a greater impulsive velocity to be imparted to a plate than a  $\frac{1}{2}$  in. thick layer of foam for a given amount of explosive. In other words, for a given impulsive

Nominal thickness (in.)	σ <sub>0</sub> (psi)	σ <sub>u</sub> (psi)	С (%)	Mn (%)	Р (%)	S (%)	Si (%)
0.064	35,900	47,300	0.071	0.41	0.008	0.021	0.001
0-098	33,800	43,000	0.046	0.25	0.007	0.019	0.001
0.173	36,800	48,000	0.086	0.34	0-009	0.023	0.003

TABLE 3. MECHANICAL AND CHEMICAL PROPERTIES OF MILD STEEL SPECIMENS ( $\rho = 0.279 \text{ lb/in.}^3$ )

Table 4. Mechanical and chemical properties of aluminum 6061-T6 specimens  $(
ho = 0.0988 \ {\rm lb/in.^3})$ 

Nominal thickness (in.)	σ <sub>0</sub> (psi)	σ" (psi)	Si (%)	Cu (%)	Mg (%)	Cr (%)
0.122	41,166	45,833	0.65	0.20	0.85	0.26
0.188	40,750	45,400	0.61	0-21	0.83	0.21
0.244	41,450		0.60	0.24	1.04	0.17

velocity, it was not possible in the present investigation to distinguish between tests obtained using foam or neoprene attenuators.



FIG. 4. Typical tensile stress-strain curves for (a) hot-rolled mild steel, and (b) aluminum 6061-T6 specimens.

### **PREVIOUS THEORETICAL WORK**

As far as the authors are aware, no analytical solutions have been obtained for rectangular plates subjected to dynamic loads which are sufficiently large to produce plastic flow of the plate material. This is hardly surprising, because the exact collapse load of a rigid, perfectly plastic rectangular plate has not been obtained but only bounded (albeit close) using the limit theorems of plasticity [11]. Cox and Morland [12] have studied the dynamic response of a rigid, perfectly plastic square plate which is simply supported around the outer edges. However, it appears that this analysis cannot be extended easily to the particular case of a rectangular plate fully clamped around the outer edges.

The elusive nature of analytical solutions to many rigid-plastic dynamic problems underlines the value of the theorems of Martin [13] which can be used to predict a lower bound to the response time and an upper bound to the permanent deflections of a structure when subjected to impulsive velocities. These theorems are very powerful because of their simplicity and the fact that they allow estimates to be made of the behavior of rigid, perfectly plastic structures which have any shape and a broad class of boundary conditions. However, in some situations, they tend to overestimate the permanent deflections by a significant amount because the influence of finite-deflections and material strain-rate sensitivity are disregarded in their formulation. Currently, the analytical procedures of Refs. [13 and 14] appear to be the only ones available which can be used to estimate the response of structures which are non-axisymmetric, such as the case studied herein, and are subjected to dynamic loads sufficient to cause extensive plastic flow of the material.

Martin [13] proved that the duration of response  $t_f$  of a rigid, perfectly plastic continuum with a time-independent density  $\rho$ , subjected to an initial velocity field  $v_i$  (i = 1, 2, 3) is given by the inequality

$$t_f \ge \frac{\int_V \rho v_i \dot{u}_i^c \, \mathrm{d}V}{\int_V \sigma_{ij}^c \dot{\varepsilon}_{ij}^c \, \mathrm{d}V} \tag{1}$$

where the superscripts c refer to any time-independent kinematically admissible set  $(\dot{u}_{i}^{c}, \dot{\varepsilon}_{ij}^{c}, \sigma_{ij}^{c})$ . Martin [13] also showed that

$$\int_{A} T_{i}^{s} u_{i}^{f} \, \mathrm{d}A \le K_{0} \tag{2}$$

where

$$K_0 = \int_V \frac{\rho}{2} v_i v_i \,\mathrm{d}V \tag{3}$$

 $u_i^f$  is the actual permanent displacement field, and  $T_i^s$  are any statically admissible surface tractions applied to an identical continuum. It is possible to recast equation (2) into the following form:

$$\delta_f \le \frac{K_0}{R^L} \tag{4}$$

where  $\delta_f$  is the permanent deflection of the actual problem at the location and in the direction of any quasistatic limit load  $R^L$  of an identical continuum [13].

In order to apply equation (1) to a rectangular plate loaded with a uniformly distributed initial impulsive velocity  $V_0$ , we select the velocity fields

$$\dot{w} = \frac{\dot{W}_0(B\tan\phi - x')}{B\tan\phi}$$
(5)

and

$$\dot{w} = \frac{\dot{W}_0(B-y)}{B} \tag{6}$$

for regions I and II, respectively, which are indicated in Fig. 5. The straight lines ab, bc, cd, de, ef, fa, bd and ea indicated in Fig. 5 are hinge lines and form a kinematically admissible collapse mechanism. If

$$\tan\phi = \sqrt{(3+\beta^2) - \beta} \tag{7}$$

then these velocity fields correspond to those used by Wood [11] for the minimum upper bound analysis of a fully clamped rectangular plate made from a rigid, perfectly plastic material and loaded with a uniformly distributed time-independent pressure. It may be



FIG. 5. Collapse mechanism of a fully clamped rectangular plate.

shown that equations (5)-(7) allow the inequality (1) to be rewritten

$$t_f \ge \frac{mV_0 B^2}{12M_0} \{ \sqrt{(3+\beta^2) - \beta} \}^2.$$
(8)

Haythornthwaite and Shield [15] and others have indicated that the largest concentrated transverse load which an arbitrary shaped fully clamped plate of uniform thickness can withstand is

$$P = 2\pi M_0 \tag{9}$$

according to the Tresca yield condition. Substituting (9) into inequality (4) yields

$$\frac{\partial_f}{H} \le \frac{\lambda\beta}{\pi} \tag{10}$$

where

$$\lambda = \frac{\rho V_0^2 L^2}{M_0}.$$
(11)

Equation (9), and therefore inequality (10), remains valid for a point load P positioned anywhere in a rectangular plate. It is clear, however, that the bound will be most accurate at the center of the plate.

Martin [13] used inequalities (1) and (4) in order to study the particular case of a simply supported circular plate loaded with a uniform impulse. The value of  $t_f$  predicted by the equality (1) is identical to the analytical result obtained by Wang [16], while the equality (4) gives a maximum permanent deflection twice the analytical value. Despite the observations of Nayfeh and Prager [17], the time bound of Martin is very close to the analytical values in all the comparisons which have so far been published in the open literature. It is noted that the permanent deflection anywhere on a stable structure must certainly be less than  $V_0 t_f$ . If the equality obtained for  $t_f$  in Ref. [13] is multiplied by  $V_0$  then a maximum permanent deflection is obtained which is only  $\frac{1}{3}$  larger than the corresponding analytical result. Clearly, this method can only be used successfully when the time bound is very close to the analytical result so that it lacks the reliability of inequality (4). If this procedure is repeated using equality (1) multiplied by  $V_0$  then it may be shown that

$$\frac{\delta_f^*}{H} = \frac{\lambda \beta^2}{12} (\sqrt{(3+\beta^2)} - \beta)^2 \tag{12}$$

where  $\delta_f^*$  is the permanent deflection predicted at the plate center.

If a procedure similar to that leading up to inequality (8) for  $t_f$  is repeated for a simply supported rectangular plate subjected to a uniformly distributed impulsive velocity  $V_0$ , then it is found that the right-hand side of this inequality is doubled. For the particular case of a square plate ( $\beta = 1$ ) the smallest value of  $t_f$  permitted by this bound agrees with the analytical result of Cox and Morland [12]. The value of  $\delta_f^*$  predicted by the equation corresponding to (12) is  $\frac{1}{3}$  larger than the maximum permanent deflection obtained in Ref. [12].

In order to improve the predictions of Martin's theorems, particularly for the displacement bound (4), Martin and Symonds [14] developed a rational mode approximation procedure for evaluating the response time and permanent deformations of impulsively loaded structures. Unfortunately, however, this method cannot be used currently for a rectangular plate since no statically admissible mode solutions for rigid perfectly plastic rectangular plates subjected to any form of dynamic loading appear to have been published in the open literature.

The numerical procedure developed by the authors of Refs. [1 and 18] has been used to study the dynamic response of a wide variety of structures. The influence of finite-deflections and actual stress-strain-strain rate material properties have been incorporated in these programmes. However, no results appear to have been published by these authors on the dynamic behavior of rectangular plates.

Lindbergh and Boyd [3] used a numerical approach in order to investigate the response of rigid, strain-hardening membranes loaded dynamically. The rectangular membranes studied in Ref. [3] have different aspect ratios to those reported herein and considerably larger  $W_0/H$  ratios than are of interest in this article.

#### DISCUSSION

The experimental values of the maximum permanent deflections  $(W_0)$  of fully clamped rectangular plates ( $\beta = 0.593$ ), which arise from uniformly distributed impulsive velocities  $(V_0)$ , are presented in Fig. 6. It is evident that  $W_0$  is related linearly to  $V_0$  for a given plate thickness *H* over the entire range of  $V_0$  examined in the current tests. However, these lines must become non-linear for values of  $V_0$  smaller than those reported herein since the straight lines, if extended beyond the experimental data, do not pass through the origins of the graphs. A similar linearity between these two parameters for a given plate thickness (H) was also observed in the experimental tests on wide beams and rectangular plates clamped at two ends which were reported in Ref. [10].

Typical permanent profiles of the mild steel and aluminum 6061-T6 rectangular plates studied herein are shown in Figs. 7–10. It is interesting to observe that these profiles bear a striking resemblance to the shape of the velocity field used by Wood [11] to calculate the minimum upper bound of a fully clamped rectangular plate made from a rigid, perfectly plastic material and loaded with a uniformly distributed static pressure. It should be remarked in passing, however, that Keil [19] observed a significant difference between the



FIG. 6.  $W_0/H$  vs.  $V_0$  for (a) mild steel, and (b) aluminum 6061-T6 rectangular plates.

strain distributions in and deformed shapes of circular plates loaded dynamically and the same plates loaded statically.

The predictions of Martin's theorems are compared in Fig. 11 with the maximum permanent deflections of the mild steel and aluminum 6061-T6 rectangular plates studied herein. In view of the previous observations concerning the final displacement fields, it is believed that the time bound  $t_f$  given by equation (8) is probably quite close to the exact value. Thus, the predictions of equation (12) should provide an upper bound to the actual deflections. The results presented in Fig. 11 indicate that the displacements predicted by equation (10) are too large even for extremely small values of  $W_0/H$ , while equation (12)







FIG. 8. Permanent profile of mild steel specimen No. 14.



FIG. 9. Permanent profile of aluminum 6061-T6 specimen No. 3.



FIG. 10. Permanent profile of aluminum 6061-T6 specimen No. 19.

gives acceptable engineering estimates of the permanent deflections up to the order of  $\frac{1}{2}$  the plate thickness. It is observed from previous experimental and theoretical work on axially restrained beams [8] and annular plates [20] that infinitesimal or bending only theories provide reasonable predictions up to the order of  $\frac{1}{2}$  the beam or plate thickness. For deflections larger than this, the favorable influence of geometry changes reduces significantly the permanent deflections below those predicted by an infinitesimal theory.

It is evident from Fig. 11 that the permanent deflections of plates made from mild steel are smaller than those made from aluminum 6061-T6. This is believed to be due principally



FIG. 11. Comparison of experimental results for mild steel and aluminum 6061-T6 rectangular plates with theoretical predictions ( $\beta = 0.593$ ).

to the influence of material strain rate sensitivity [4]. When  $\lambda = 200$ , the permanent deflection of a fully clamped mild steel plate is 2.575*H* compared with 3.175*H* for aluminum 6061-T6 plate. This reduction in deflection, is slightly less than that observed in Ref. [10] for rectangular plates clamped at two ends. In Ref. [10], the maximum permanent deflection of an aluminum 6061-T6 rectangular plate is 3.175*H* when  $\lambda \simeq 66$  with a corresponding value of 2.35*H*, approximately, for a mild steel plate.

The deflection parameter  $W_0/H$  and impulse parameter  $\lambda$  allow a single curve to be drawn through all the experimental results for each material over the entire range of  $\lambda$  given in Fig. 11. In view of the different thicknesses of the rectangular plates, material strain-hardening does not appear to have any important influence on the final response.

### CONCLUSIONS

An experimental investigation was undertaken in order to study the behavior of fully clamped rectangular plates when subjected to uniformly distributed impulsive velocities. The total energy of the dynamic loads was sufficiently large to cause plastic flow of the plate material and maximum permanent deflections from 0.2 to nearly seven times the corresponding plate thicknesses. All the rectangular plates had the same aspect ratio (B/L = 0.593) but various thicknesses (H), and were made from either hot rolled mild steel or aluminum 6061-T6. The permanent deformed profiles of the plates are similar to the shape of the velocity field used by Wood [11] for calculating the minimum upper bound to the collapse pressure of a fully clamped rectangular plate loaded with a uniformly distributed time-independent pressure. It is observed that a modification of the bending only prediction of Martin [13] provides adequate engineering estimates of the maximum permanent deflections up to the order of  $\frac{1}{2}$  of the corresponding plate thickness. For larger deflections, it is necessary to include the influence of geometry changes; and in the case of mild steel, material strain-rate sensitivity as well.

Acknowledgements—The work reported herein was supported by O.N.R. (Contract No. N00014-67-A-0204-0032), the Sloan Foundation and the Department of Naval Architecture and Marine Engineering at M.I.T. The authors are also indebted to Professor A. H. Keil for his encouragement during this work. The authors wish to take this opportunity to express their appreciation to Dr. J. W. Leech, F. Merlis and O. E. Wallin of the Aeroelastic Laboratory for their kind cooperation and to Professor E. A. Witmer for permission to use the blast chamber in which the tests were conducted.

### REFERENCES

- [1] J. W. LEECH, E. A. WITMER and T. H. H. PIAN, A numerical calculation technique for large elastic-plastic transient deformations of thin shells. AIAA Jl 6, 2352 (1968).
- [2] T. A. DUFFEY and S. W. KEY, Experimental-theoretical correlations of impulsively loaded clamped circular plates. *Exp. Mech.* 9, 241 (1969).
- [3] C. LINDBERGH and D. E. BOYD, Finite, inelastic deformations of clamped shell membranes subjected to impulsive loadings. AIAA Jl 7, 228 (1969).
- [4] P. S. SYMONDS, Survey of Methods of Analysis For Plastic Deformation of Structures Under Dynamic Loading, Brown University Report BU/NSRDC/1-67 (1967).
- [5] B. RAWLINGS, The present state of knowledge of the behavior of steel structures under the action of impulsive loads. Trans. Instn Engrs Aust. CE5, 89 (1963).
- [6] J. S. HUMPHREYS, Plastic deformation of impulsively loaded straight clamped beams. J. appl. Mech. 32, 7 (1965).
- [7] A. L. FLORENCE and R. D. FIRTH, Rigid-plastic beams under uniformly distributed impulses. J. appl. Mech. 32, 481 (1965).
- [8] P. S. SYMONDS and N. JONES, Impulsive Loading of Fully Clamped Beams with Finite Plastic Deflections and Rate Sensitivity, Brown University Report.
- [9] A. L. FLORENCE, Circular plate under a uniformly distributed impulse. Int. J. Solids Struct. 2, 37 (1966).
- [10] N. JONES, R. N. GRIFFIN and R. E. VAN DUZER, An Experimental Study into the Dynamic Plastic Behavior of Wide Beams and Rectangular Plates, M.I.T. Report No. 69–12 (1969).
- [11] R. H. WOOD, Plastic and Elastic Design of Slabs and Plates. Ronald Press (1961).
- [12] A. D. Cox and L. W. MORLAND, Dynamic plastic deformations of simply-supported square plates. J. Mech. Phys. Solids 7, 229 (1959).
- [13] J. B. MARTIN, Impulsive loading theorems for rigid-plastic continua. Proc. Am. Soc. civ. Engrs 90, 27 (1964).
- [14] J. B. MARTIN and P. S. SYMONDS, Mode approximations for impulsively loaded rigid-plastic structures. Proc. Am. Soc. civ. Engrs. 92, 43 (1966).
- [15] R. M. HAYTHORNTHWAITE and R. T. SHIELD, A note on the deformable region in a rigid-plastic structure. J. Mech. Phys. Solids 6, 127 (1958).
- [16] A. J. WANG, The permanent deflection of a plastic plate under blast loading. J. appl. Mech. 22, 375 (1955).
- [17] A. NAYFEH and W. PRAGER, Response time of impulsively loaded structures. Proc. Am. Soc. civ. Engrs 95, 813 (1969).
- [18] L. MORINO, J. W. LEECH and E. A. WITMER, An improved numerical calculation technique for large elasticplastic transient deformations of thin shells, Parts 1 and 2. J. appl. Mech. to appear.
- [19] A. H. KEIL, Problems of Plasticity in Naval Structures: Explosive and Impact Loading, *Proc. Second Symp.* on Naval Structure Mech. Pergamon Press (1961).
- [20] N. Jones, Finite deflections of a rigid-viscoplastic strain-hardening annular plate loaded impulsively. J. appl. Mech. 35, 349 (1968).

(Received 8 December 1969)

Абстракт—Предпринимаются экспериментальные исследозания с целью изучения поведения полно защемленных прямоугольных пластинок, подверженных действию равномерно распределенных импульсивных скоростей. Полная энергия динамических нагрузок оказывается достаточно большой, чтобы вызвать пластическое течение материала пластинки и максимальны остаточные прогибы, величина которых колебается от 0, 2 до семикратной соответствующей толшины пластинки. Все прямоугольные пластинки обладают таким же самым отношением сторон (B/L = 0,593), но разной (H). Они изготовлены либо из горачекатанной мягкой стали, или из алюминия 6061–Т6. Остаточные деформированные очертания пластинок подобны по форме нолю скоростей, использованным Уудом (11) для расчета минимального верхнего предела относительно давления разрушения полно защемленной прямоугольной пластинки, нагруженной равномерно распределенным, независимым от времени, давлением. Наблюдается, что модификация предсказания только изгиба, предложенного Мартином (13), дает соответствующей толщины пластинки. Для больших прогибов вплоть до порядка половины соответствующей толщины пластинки. Для больших прогибов надо учесть влияние геометрических изменений, но в случае мягкой стали, также, чувствительность к скорости деформации материала.